

Calculators and Mobile Phones are not allowed.

1. (6 points) Evaluate the following integrals:

a) $\int_0^1 (2x - 1)^7 dx$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5\left(\frac{x}{2}\right) \cos x dx$

c) $\int \frac{1}{x^2} \left(1 - \frac{1}{x}\right)^{13} dx$

d) $\int \sec^2(x - 1) \tan(x - 1) dx$

2. (3 points) Evaluate

$$\frac{d}{dx} \left(\int_0^{\tan x} \sqrt{s^2 + 1} ds \right)$$

3. (4 points) Let $f(x) = 2x - 7$

- a) Find all values c that satisfy the conclusion of the Mean Value Theorem (M.V.T.) on $[1, 5]$.
b) Use the M.V.T. for definite integrals to find f_{av} and find all values x where it is attained.

4. (9 points) Let $f(x) = \frac{x + 2}{x^2 + x - 2}$

- a) Find the vertical and horizontal asymptotes (if any).
b) Show that $f'(x) = \frac{-1}{(x - 1)^2}$ is f differentiable at $x = -2$.
c) Find the intervals on which f is increasing and those on which f is decreasing.
d) Show that $f''(x) = \frac{2}{(x - 1)^3}$. Find the intervals on which f is concave upward and those on which f is concave downward and find the points of inflection (if any).
e) Sketch the graph of f .

5. (3 points) Let ABC be a right triangle at B with a fixed area $A = 50 \text{ cm}^2$. Find the dimensions of the sides AB and BC so that their sum is minimum.

